THE DETERMINATION OF THE 'DIFFUSION COEFFICIENTS' AND THE STELLAR WIND VELOCITIES FOR X–RAY BINARIES.

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Abstract

The distribution of neutron stars (NS's) is determined by stationary solution of the Fokker–Planck equation. In this work using the observed period changes for four systems: Vela X–1, GX 301–2, Her X–1 and Cen X–3 we determined D, the 'diffusion coefficient',– parameter from the Fokker–Planck equation. Using strong dependence of D on the velocity for Vela X–1 and GX 301–2, systems accreting from a stellar wind, we determined the stellar wind velocity. For different assumptions for a turbulent velocity we obtained $V = (660 - 1440)\, km \cdot s^{-1}$. It is in good agreement with the stellar wind velocity determined by other methods.

We also determined the specific characteristic time scales for the 'diffusion processes' in X-ray pulsars. It is of order of 200 sec for wind-fed pulsars and 1000-10000 sec for the disk accreting systems.

Keywords: accretion:neutron stars–stars:stellar wind–stars.

1 Introduction

The most precisely determined characteristic for accreting neutron stars (NS's) is their period. Thus using observations of the period we can determine different properties of the observed object.

Period changes show fluctuations. These fluctuations were discussed in de Kool & Anzer (1993). The authors determined noise level in these systems and characteristic time scales. Using these results we can estimate diffusion coefficients in the Fokker–Planck equation (see below) and the stellar wind velocities (only for the wind-accreting systems).

During accretion the angular momentum of plasma is transfered to the NS. But the process of the momentum transfer is not stationary. The transfered angular momentum fluctuates and therefore the period changes of the NS will also show fluctuations.

Table 1:

	p, sec	p_{orb}, sec	L, erg/sec	$\mu/10^{30}\,Gs\cdot cm^3$	$t_{su\,observ}$, yrs	$t_{su\,min}$, yrs
Vela X–1	283	$7.7\cdot 10^5$	$1.5\cdot 10^{36}$	3	3000	3000
GX 301–2	696	$3.6\cdot 10^6$	10^{37}	120	>100	100
Her X–1	1.24	$1.5\cdot 10^5$	10^{37}	0.6	$3\cdot 10^5$	8000
Cen X–3	4.84	$1.8\cdot 10^5$	$5\cdot 10^{37}$	5.7	3400	600

Processes with fluctuations are well known (see for example Haken (1978)). Some applications of stochastic processes in astrophysics especially in accreting systems were discussed in Lipunov (1987),Hoshino & Takeshima (1993) and Lipunov (1992).In Hoshino & Takeshima (1993) the authors, using simple models of MHD turbulence, try to explain aperiodic changes in X–ray luminosity of X–ray pulsars. Luminosity fluctuations are explained as the result of density fluctuations due to turbulence in the plasma flow. The authors used 2D model for accretion disk and 3d model for the wind accreting systems. Detailed exploration of this question is very difficult in both: theoretical and observational ways (there is no good theory of MHD turbulence and the resolution of modern equipment of satellites is not high enough for power spectra of X–ray pulsars (see Hoshina & Takeshima 1993). But detailed exploration of the density fluctuations will help to understand period fluctuations. It will be very interesting to compare X–ray luminosity fluctuations with fluctuations of the period of the NS.

There are different methods of describing of these processes. In this work we use differential equations for the distribution function. For the frequency changes we can write the Langeven equation, which describes the process with fluctuations:

$$\frac{d\omega}{dt} = F(\omega) + \Phi(t) \tag{1}$$

Here, $F(\omega)$ –constant angular momentum. For $F(\omega)$ in the most general form we can write (Lipunov 1982):

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F(\omega )\cdot I=\left\{
 \begin{array}
 & \dot{M}\eta _k \Omega R_G^2-k_t\frac {\mu ^2}{R_c^3},
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\qquad  {\text wind\, accretion}\\
& \dot{M}\sqrt{GMR_d}-k_t\frac {\mu ^2}{R_c^3},
\qquad {\text accretion\, disk},\\
\end{array}
\right.
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where R_d–disk radius, k_t and η_k–dimensionless parameters ($k_t \approx 1$, $\eta_k \approx 1$), I–moment of inertia of the NS, $M-$ mass of the NS, Ω–orbital frequency of the system, R_G- radius of gravitational capture ($R_G = \frac{2GM}{v^2+v_{orb}^2}$, we include here the orbital velocity) and R_c–corotational radius, $R_c = \left(\frac{GM}{\omega^2}\right)^{1/3}$.

We assume that the 'force' is conservative and in this case we can write $F(\omega)$ in the form: $F(\omega) = -\nabla_\omega V$, where V is a scalar potential. Φ is a fluctuating moment, i.e. $< \Phi(t) >= 0$, (Lipunov 1987).

The distribution of frequency, ω , is described by the distribution function $f(\omega)$. This function satisfies the Fokker–Planck equation (Haken 1978):

$$\frac{df}{dt} = \frac{df\, F(\omega)}{d\omega} + D\frac{d^2 f}{d\omega^2}, \tag{2}$$

where D is the 'diffusion coefficient', which is determined by the correlation of the stochastic force Φ :

$$< \Phi(t)\Phi(t') >= 2D\delta(t-t') \tag{3}$$

Stationary solution of the eq.(2) is the following:

$$f(\omega) = N \exp\left(-V(\omega)/D\right) \tag{4}$$

where N is determined from the normalisation condition:

$$\int_{-\infty}^{\infty} f(\omega)\, d\omega = 1 \tag{5}$$

Using expression for $V(\omega)$ from Lipunov (1992) we can write D in the form:

$$D = \frac{k_t \mu^2}{3GMI} \cdot \frac{\omega^3}{\gamma} \tag{6}$$

Table 2:

	A	L_{max}
Vela X–1	-9.1	$10^{36.8}$
GX 301–2	-8.5	10^{37}

where k_t–constant parameter ($k_t \approx 1$) , I–moment of inertia of the NS and γ is evaluated as $\gamma \approx \frac{t_{su}}{\Delta t}$, here t_{su}–time of spin-up and Δt –the characteristic time for period changes (see for details Lipunov (1987) or Lipunov (1992)).

Using eq.(6) in 2.1 we shall determine the value of the 'diffusion coefficient', D. With these D in 2.2 we shall make the estimates of the stellar wind velocities for Vela X–1 and GX 301–2.

2 Results.

2.1 Determination of the 'diffusion coefficient'.

At first we shall estimate D, using eq. (6). In this equation all variables, except Δt , are known (in principle). Their values taken from Lipunov (1992) are shown in table 1.

Nagase (1992) gives results of $GINGA$ observations of the cyclotron lines in X–ray pulsars. For Vela X-1 and Her X-1 values of magnetic field, B, coincides with the values μ that we used when radii of NS's are $10\,km$ and $6\,km$ correspondently. The value for B obtained from observations for Vela X-1, $B = 2.3 \cdot 10^{12}\,Gs$, coincides quite well with the assumption that there is no stable accretion disk in this pulsar.

Characteristic time Δt for the wind–accreting systems can be determined from the equation:

$$\Delta t \approx 1.7 \cdot 10^4 \alpha^{-2} 10^{2(A+8.5)} L_{37}^{-\frac{12}{7}} \mu_{30}^{-\frac{4}{7}} sec, \qquad (7)$$

where α – a fraction of the specific angular momentum of the Kepler orbit at the magnitospheric radius, A–noise level (see table 2) (de Kool & Anzer 1993).

In eq.(6) for t_{su} we must use minimum values. These times were calculated using equations for pure spin-up from Lipunov (1992), $t_{su\,min}$ are shown in

table 1.

We took $k_t = 1/3, M = 1.5\, M_\odot, I = 10^{45}\, g \cdot cm^2$. From eq. (7) we can get Δt for Vela X–1 and GX 301–2. For Her X–1 and Cen X–3, Δt is determined from the graph in de Kool & Anzer (1993) (see table 3). So we can write equation for D in the form:

$$D = 5.55 \cdot 10^{-19} \mu_{30}^2 \omega^3 \gamma_6^{-1} I_{45}^{-1} \left(\frac{M}{1.5 M_\odot} \right)^{-1} \quad s^{-3}\,. \tag{8}$$

Values of D for four systems are shown in table 3.

From the theory of diffusion we can write:

$$D = \omega_{char} \dot{\omega}\,, \tag{9}$$

where ω_{char} is the characteristic length in the frequency space.

For characteristic time in this space we can write:

$$t_{char} = \omega_{char}/\dot{\omega} = \frac{D p^4}{4\pi^2 \dot{p}^2} \tag{10}$$

We can give a physical interpretation for t_{char} for wind-fed pulsars as a characteristic time of the momentum transfer:

$$\frac{R_G}{v_{sw}} = 400 \left(\frac{M}{1.5 M_\odot} \right) \left(\frac{v_{sw}}{10^8 cm/s} \right)^{-3} \qquad sec \tag{11}$$

For $v_{sw} = 1178\, km/s$ we obtain $\frac{R_G}{v_{sw}} = 245\, sec$ and for $v_{sw} = 1281\, km/s$ we obtain $\frac{R_G}{v_{sw}} = 200\, sec$. These velocities are close to v_{sw} obtained using estimates of the 'diffusion coefficient'.

These characteristic time scales are also shown in table 3.

2.2 Determination of the stellar wind velocity.

Fluctuating moment Φ (see eq.(1)) can be estimated as:

$$\left(\frac{\dot{M}\, v_t\, R_t}{I} \right)$$

So for D we can write the equation which differs from eq.(6) (Lipunov & Popov 1995):

Table 3:

	Δt, sec	γ	D, sec^{-3}	v_{sw}, km/s, ($v_t = 0.1 a_s$)	v_{sw}, km/s, ($v_t = a_s$)	t_{char}, sec
Vela X–1	$1.5 \cdot 10^4$	$6.3 \cdot 10^6$	$8.7 \cdot 10^{-24}$	848	1442	200
GX 301–2	$1.1 \cdot 10^3$	$2.9 \cdot 10^6$	$2 \cdot 10^{-21}$	656	1120	245
Her X–1	$4 \cdot 10^3$	$6.0 \cdot 10^7$	$4 \cdot 10^{-19}$	—	—	960
Cen X–3	$2 \cdot 10^4$	$9.5 \cdot 10^5$	$4.1 \cdot 10^{-17}$	—	—	8500

$$D = \frac{1}{2}\left(\frac{\dot{M}\, v_t\, R_t}{I}\right)^2 \frac{R_G}{v_{sw}}, \tag{12}$$

where v_t–turbulent velocity, R_t–characteristic scale of the turbulence. This scale is of order of radius of gravitational capture, R_G:

$$R_t \approx R_G = \frac{2GM}{v_{sw}^2} \approx 4 \cdot 10^{10} v_8^{-2} \left(\frac{M}{1.5\, M_\odot}\right) cm, \tag{13}$$

where $v_8 = \frac{v}{10^8\, cm/s}$.

The turbulent velocity is less or equal to the sound speed,a_s (in opposite case a great bulk of energy will dissipate in the form of shock waves), that's why we can write:

$$v_t = \eta \cdot a_s, \qquad \eta \le 1 \tag{14}$$

where $a_s = ((5\,RT)/(3\,\mu))^{1/2} = 1.18{\cdot}10^6 T_4^{1/2} \mu^{-1/2}\, cm/s$ and $T_4 = T/(10\,000K)$.

From eq. (12) we can get:

$$D = 4.38 \cdot 10^{-23} \dot{M}_{16}^2 \eta^2 T_4 \mu^{-1} v_8^{-7} I_{45}^{-2} \left(\frac{M}{1.5\, M_\odot}\right) s^{-3}\,, \tag{15}$$

here $\dot{M} = \frac{\dot{M}}{10^{16}\, g/s}$, $\quad I_{45} = \frac{I}{10^{45}\, g{\cdot}cm^2}$.

As we see there is a strong dependence of D on v. So we can evaluate v(in this case it is the stellar wind velocity, v_{sw}):

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\begin{equation}
\begin{array}
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& v_{sw}=1700\cdot D_{-24}^{-1/7}\dot{M} _{16}^{2/7}\eta ^{2/7}T_4^{1/7}
\mu ^{-1/7}\\
& I_{45}^{-2/7}\left(\frac M{1.5\, M_\odot }\right)^{2/7} \, km/s.\\
\end{array}
\end{equation}
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Values of v_{sw} are shown in table 3.

3 Conclusions

Obtained values of v_{sw} coincide well with the characteristic value of this quantity for supergiants of early spectral classes: $v_{sw} \approx (600-3000)\,km/s$ (de Jager 1980). We also use well known equation for a terminal velosity of a stellar wind which is well confirmed by observations:

$$v_\infty \approx 3 \cdot v_{esc}, \tag{16}$$

where $v_{esc} = (\frac{2GM}{R_*^2})^{1/2}$ is the escaping velocity on the surface of the star, $r = R_*$.We use this dependence and equation (de Jager 1980):

$$v(r) = V_\infty (1 - \frac{R_*}{r})^\alpha \tag{17}$$

where $\alpha \approx (0.35 - 0.5)$.

For optical component of GX 301–2 we have: $M = 35\,M_\odot, R = 43\,R_\odot$ and $e = 0.47$ (Watson et al 1982). We took $r = r_{min} \approx 2\,R_*$, because in our calculations we used maximum luminosity, i.e. luminosity in periastr. For these values we have: $v_{esc} \approx 560\,km/s$ and for v_∞ we have $v_\infty \approx 1680\,km/s$ (for ρLeo, B1 Iab, in de Jager (1980) we can find $v_{sw} = 1580\,km/s$).

From eq. (18) we get: $v_{sw}(r = 2\,R_*, \alpha = 0.5) \approx 1180\,km/s$. It coincides well with our maximum value $\approx 1150\,km/s$ (see table 3). For Vela X–1 there are such estimates (Haberl 1991): $v_\infty = 1700\,km/s, r = 1.7\,R_*$ and $\alpha = 0.35$. We get: $v_{sw} \approx 1250\,km/s$. It also coincides with our estimates of the maximum stellar wind velocity: $\approx 1450\,km/s$. With different assumptions for the turbulent velocity (for example $v_t = \frac{1}{3}a_s$) we can get excellent coincidence of our results with the stellar wind velocities estimated above using other techniques.

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